

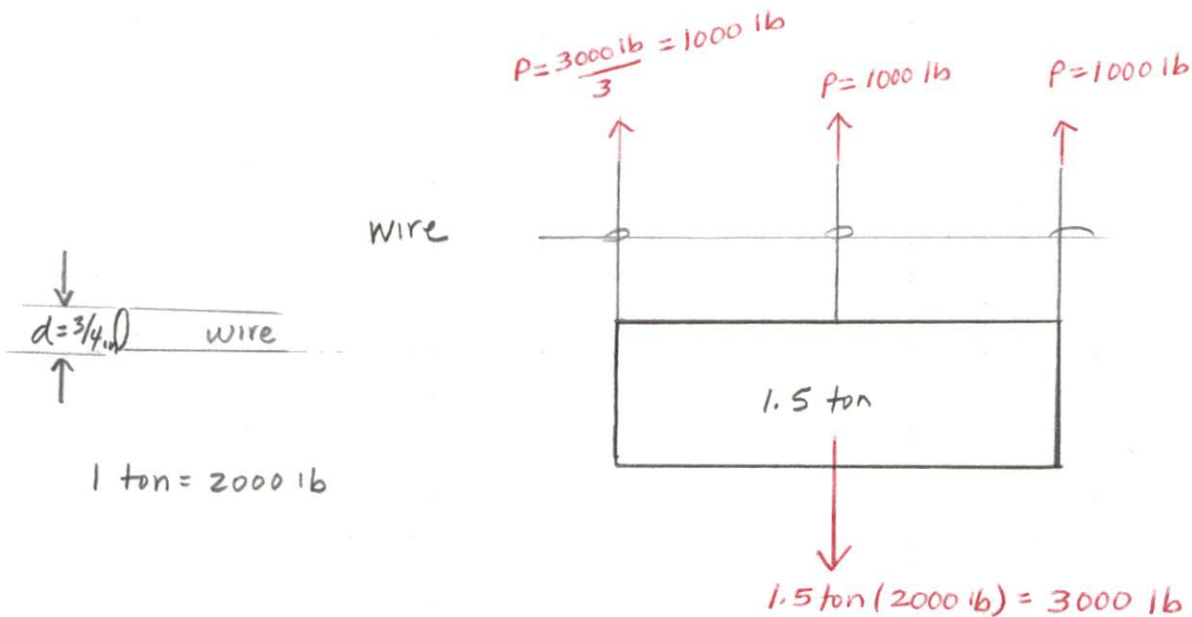
Chapter 9 - Problems

Note: All SI Unit Problems changed to US Customary Units

9-1

A 1.5-ton crate is hoisted by three steel wires. Each wire is $\frac{3}{4}$ in. in diameter and each carries one-third of the load. Determine the stress in the wires.

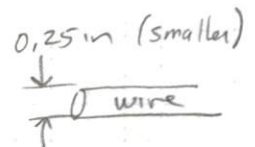
Note: Book Prob. uses $\frac{1}{4}$ in.



Normal Stress

$$\sigma = \frac{P}{A} = \frac{1000 \text{ lb}}{\frac{\pi (0.75 \text{ in.})^2}{4}} = \underline{\underline{2264 \text{ psi}}}$$

Compare to a $\frac{1}{4}$ in. diameter wire:



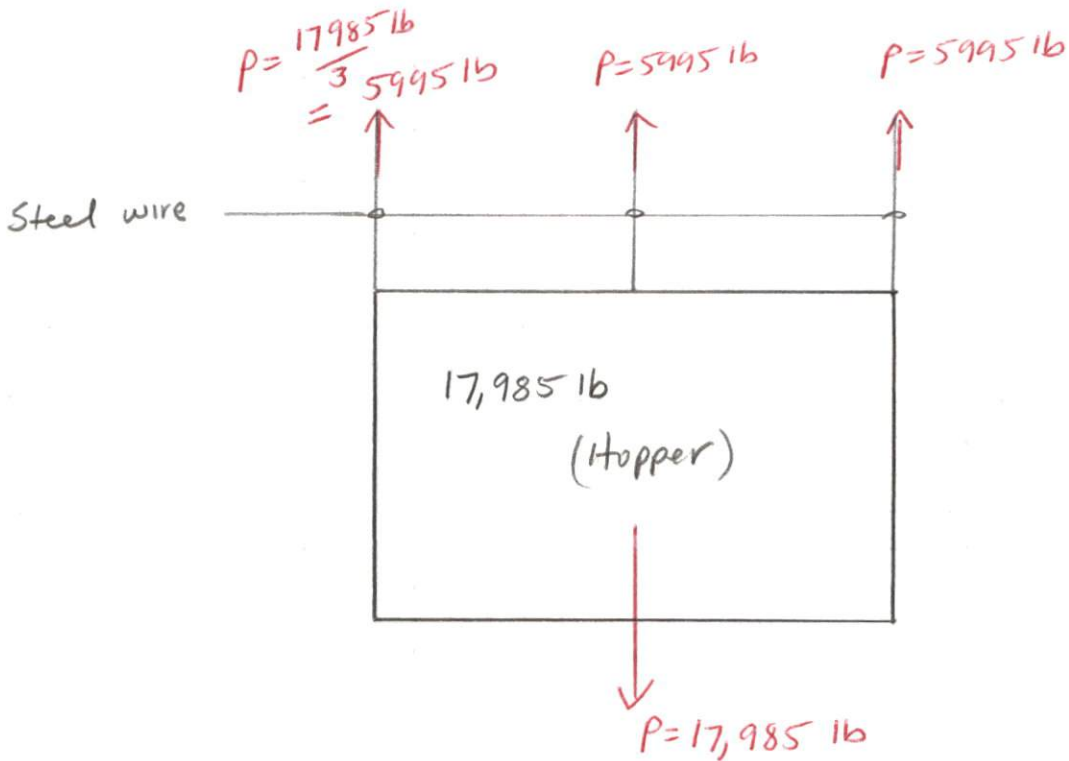
$$\sigma = \frac{P}{A} = \frac{1000 \text{ lb}}{\frac{\pi (0.25)^2}{4}} = \underline{\underline{20,370 \text{ psi}}}$$

The normal stress is much larger in the smaller diameter wire. Imagine the diameters of a Bridge Cable!

9-2

An 17,985 lb hopper is supported by three steel wires. Each wire is 0.6 in. in diameter and each carries one-third of the load. Determine the stress in the wires.

Solution.



$$d = 0.6 \text{ in} \downarrow \text{O} \text{ wire}$$

Normal stress

$$\begin{aligned} \sigma &= \frac{P}{A} = \frac{5995 \text{ lb}}{\frac{\pi (0.6 \text{ in.})^2}{4}} \\ &= 21,203 \text{ psi} \end{aligned}$$

To compare with textbook, convert to MPa

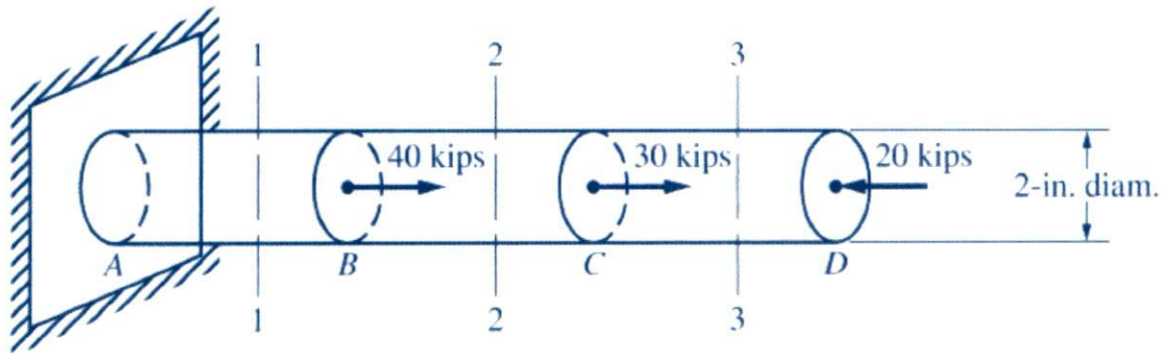
$$21,203 \text{ psi} = 146 \text{ MPa} \quad (\text{slight difference due to rounding})$$

Book ans

$$\sigma = 151 \text{ MPa}$$

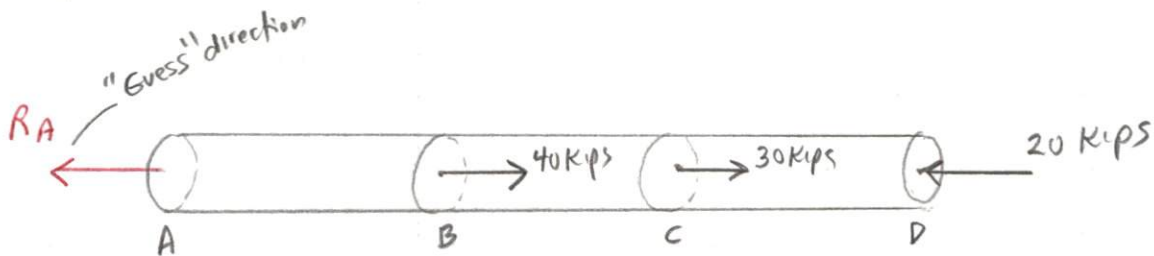
Refer to Figs. P9-3 to P9-5. Plot the internal axial force diagram and determine the normal stresses in segments AB, BC, and CD of each member due to the axial loads shown.

9-3



Solution.

Step 1. FBD Entire Rod



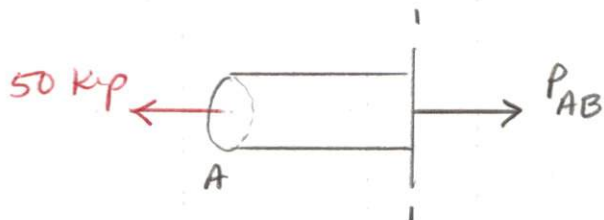
FBD- Entire Rod

Equilibrium Equations

$$[\sum F_x = 0] \quad -R_A + 40 \text{ kips} + 30 \text{ kips} - 20 \text{ kips} = 0$$

$$R_A = 70 \text{ kips} - 20 \text{ kips} = 50 \text{ kips} \leftarrow$$

Step 2. Section Rod @ 1-1, sketch FBD of Left Portion of Section 1-1



FBD- Left Portion of Section 1-1

Equilibrium Equations

$$[\sum F_x = 0]$$

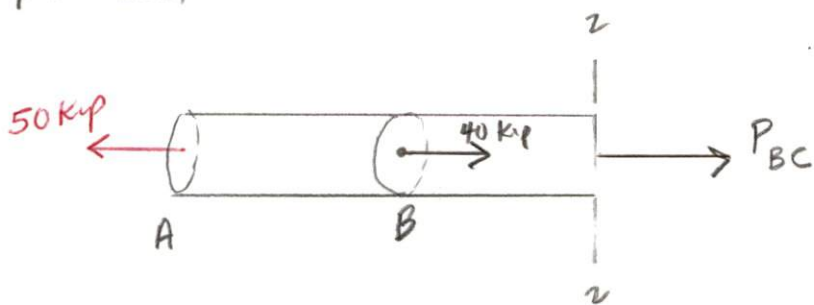
$$-50 \text{ kips} + P_{AB} = 0$$

$$P_{AB} = \underline{\underline{50 \text{ kips (T)}}$$

"out of the section" is Tension (T)

"into the section" is Compression (C)

Step 3. Section 2-2



Equilibrium Equations

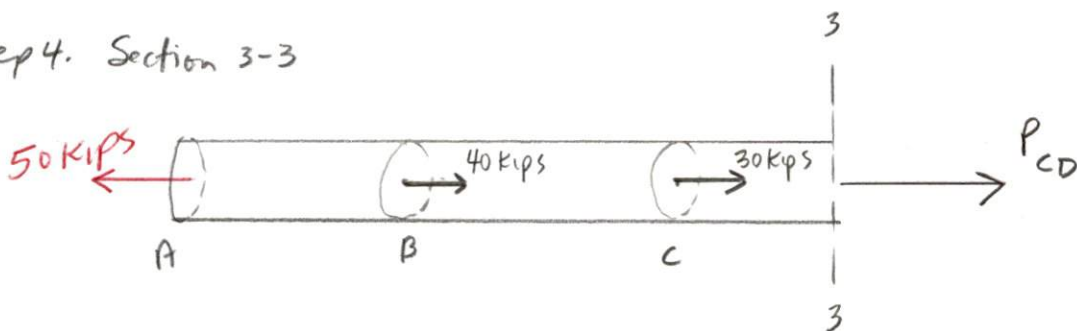
$$(\sum F_x = 0)$$

$$-50 \text{ kips} + 40 \text{ kips} + P_{BC} = 0$$

$$P_{BC} = \underline{\underline{10 \text{ kips (T)}}}$$

FBD - Left-Portion Section 2-2

Step 4. Section 3-3



FBD - Left-Portion of Section 3-3

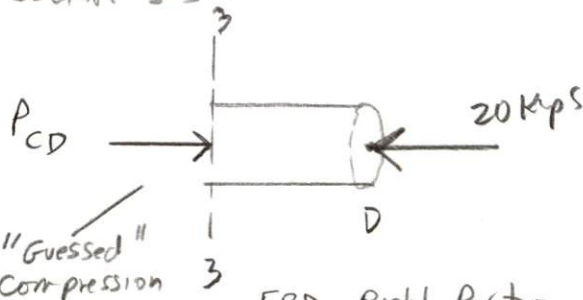
Equilibrium Equations

$$(\sum F_x = 0) \quad -50 \text{ kips} + 40 \text{ kips} + 30 \text{ kips} + P_{CD} = 0$$

$$P_{CD} = 50 \text{ kips} - 70 \text{ kips} = -20 \text{ kips (T)}$$

and $P_{CD} = 20 \text{ kips (C)}$

OR, use Right-Portion of Section 3-3



Equilibrium Equations

$$(\sum F_x = 0)$$

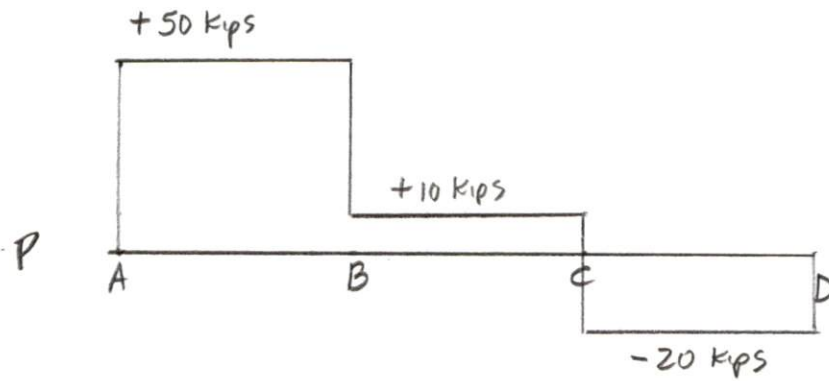
$$P_{CD} - 20 \text{ kips} = 0$$

$$P_{CD} = \underline{\underline{20 \text{ kips (C)}}}$$

"Gussed" correctly

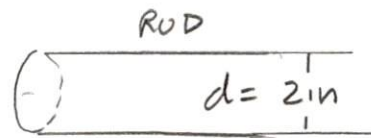
FBD - Right-Portion of Section 3-3

Internal Axial Force Diagram



Normal Stress

$$\sigma = \frac{P}{A}$$

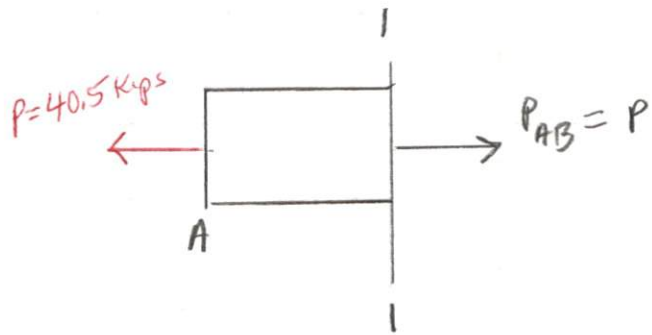
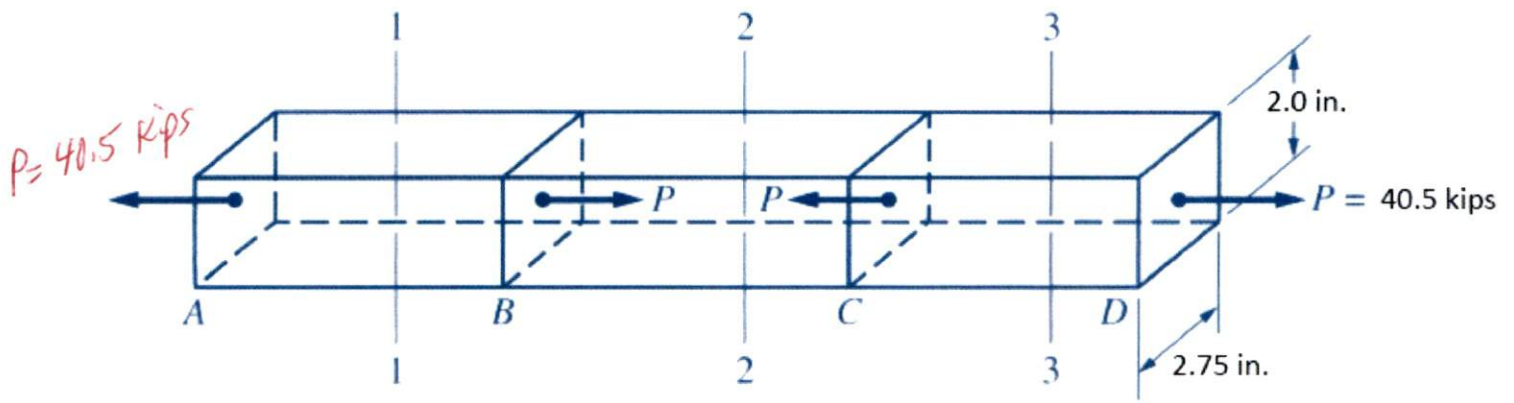


$$A = \frac{\pi d^2}{4} = \frac{\pi (2 \text{ in})^2}{4} = 3.14 \text{ in}^2$$

$$\sigma_{AB} = \frac{+50 \text{ kips}}{3.14 \text{ in}^2} = +15.92 \text{ ksi (T)}$$

$$\sigma_{BC} = \frac{+10 \text{ kips}}{3.14 \text{ in}^2} = +3.18 \text{ ksi (T)}$$

$$\sigma_{CD} = \frac{-20 \text{ kips}}{3.14 \text{ in}^2} = -6.37 \text{ ksi (C)}$$

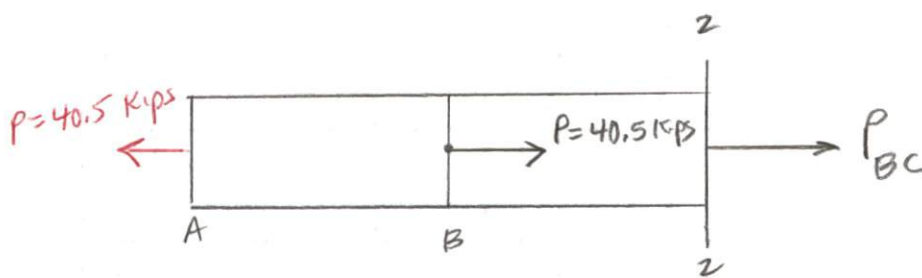


Equilibrium Equations

$$[\sum F_x = 0] \quad -40.5 \text{ kip} + P_{AB} = 0$$

$$P_{AB} = 40.5 \text{ kip (T)}$$

FBD - Left-Portion section 1-1



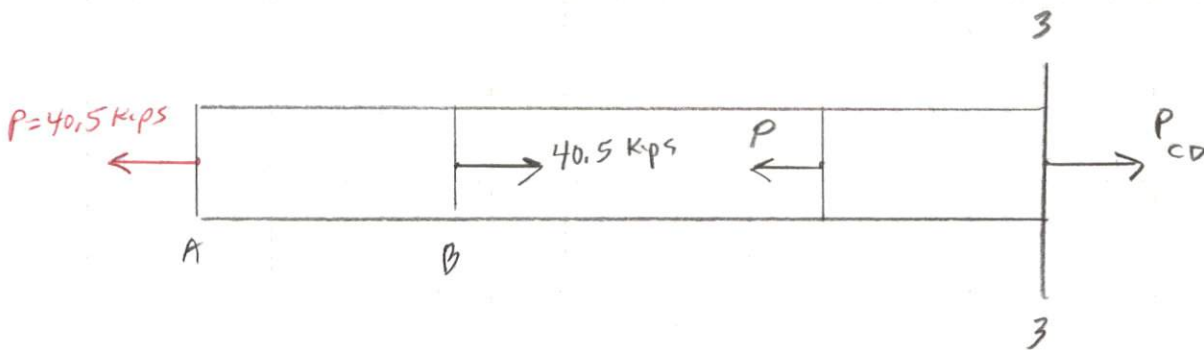
Equilibrium Equations

$$[\sum F_x = 0]$$

$$-40.5 \text{ kips} + 40.5 \text{ kips} + P_{BC} = 0$$

$$P_{BC} = 0$$

FBD - Left-Portion section 2-2



FBD - Left-Portion section 3-3

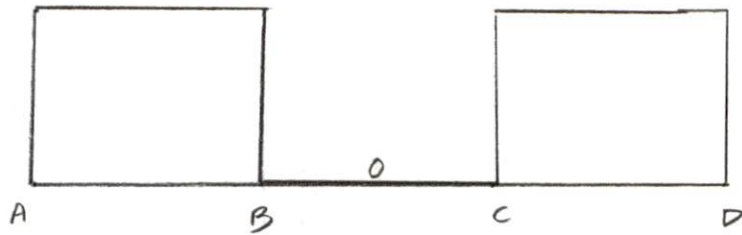
Equilibrium Equations

$$[\sum F_x = 0] \quad -40.5 \text{ kips} + 40.5 \text{ kips} - P + P_{CD} = 0$$

$$P_{CD} = P = 40.5 \text{ kips}$$

Internal Axial Force Diagram

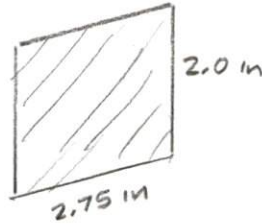
$$+P = +40.5 \text{ Kips}$$



$$+P = 40.5 \text{ Kips}$$

Cross-sectional Area

$$A = (2.75 \text{ in})(2.0 \text{ in}) \\ = 5.5 \text{ in.}^2$$



Normal stress

$$\sigma = \frac{P}{A}$$

$$\sigma_{AB} = \sigma_{CD} = \frac{40.5 \text{ Kips}}{5.5 \text{ in.}^2} = \underline{\underline{7.36 \text{ Ksi (T)}}}$$

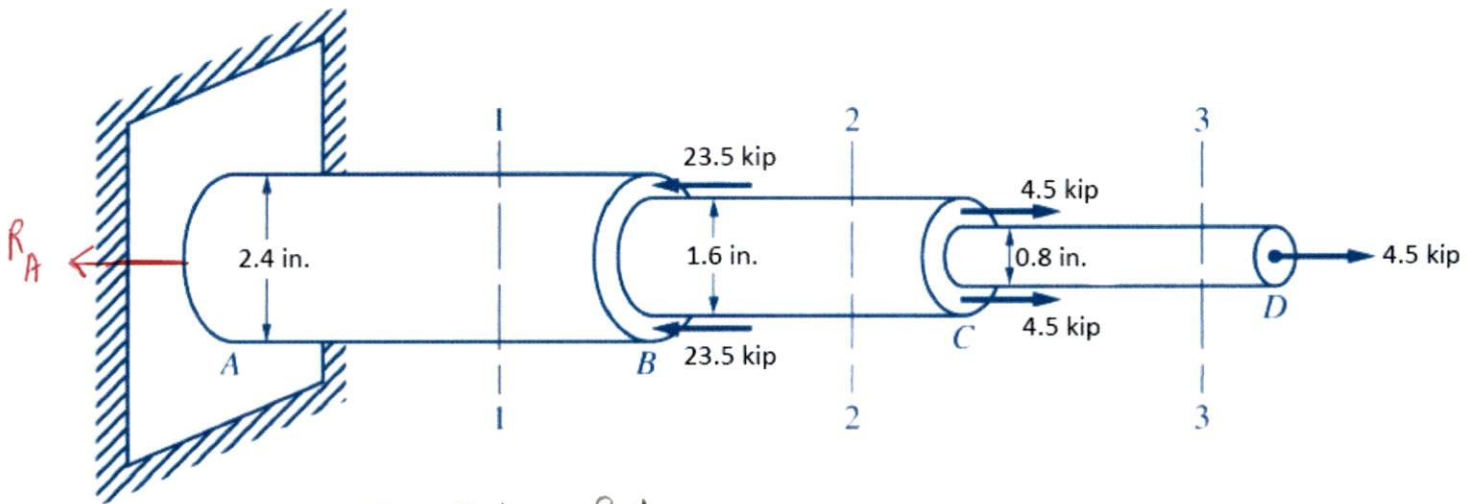
$$\sigma_{BC} = 0$$

Check,

$$7.36 \text{ Ksi} = 50.74 \text{ MPa} \checkmark$$

Book Ans

$$\sigma_{AB} = \sigma_{CD} = 51.4 \text{ MPa (T)}$$



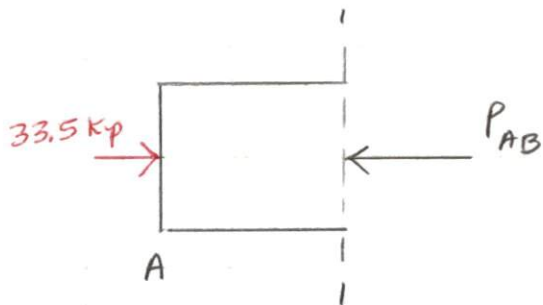
FBD- Entire Rod

Equilibrium Equations

$$[\sum F_x = 0] \quad -R_A - 2(23.5 \text{ kips}) + 2(4.5 \text{ kips}) + 4.5 \text{ kip} = 0$$

$$R_A = -47 \text{ kips} + 13.5 \text{ kips} = -33.5 \text{ kip} \leftarrow$$

$$\text{and } R_A = 33.5 \text{ kip} \rightarrow$$

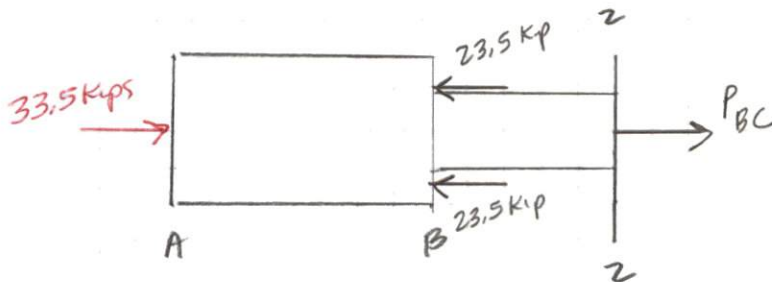


FBD- Left Portion Section 1-1

Equilibrium Equations

$$[\sum F_x = 0] \quad 33.5 \text{ kip} - P_{AB} = 0$$

$$P_{AB} = 33.5 \text{ kip (C)}$$



FBD- Left Portion Section 2-2

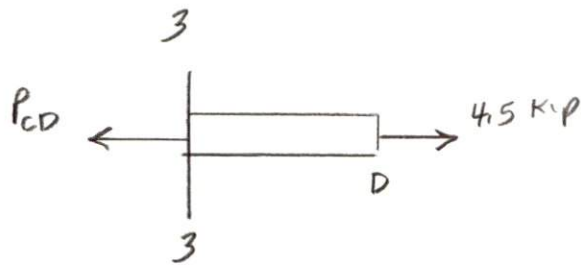
Equilibrium Equations

$$[\sum F_x = 0]$$

$$33.5 \text{ kips} - 2(23.5 \text{ kips}) + P_{BC} = 0$$

$$P_{BC} = 47 \text{ kips} - 33.5 \text{ kips}$$

$$= 13.5 \text{ kips (T)}$$



FBD- Right-portion Section 3-3

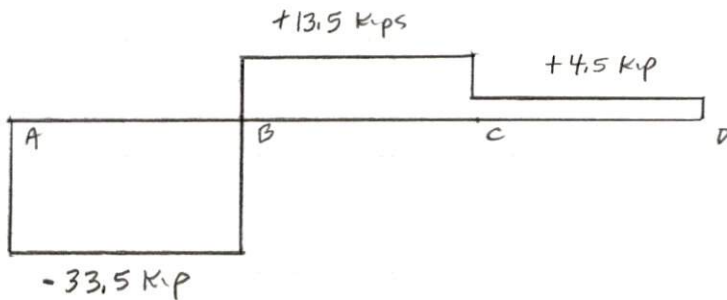
Equilibrium Eqns

$$(\sum F_x = 0)$$

$$-P_{CD} + 4.5 \text{ k-p} = 0$$

$$P_{CD} = 4.5 \text{ k-p (T)}$$

Internal Axial Force Diagram



Normal Stress

$$\sigma = \frac{P}{A}$$

$$\sigma_{AB} = \frac{-33.5 \text{ kips}}{\frac{\pi (2.4 \text{ in})^2}{4}} = -7.4 \text{ ksi (C)}$$

51 MPa

$$\sigma_{BC} = \frac{+13.5 \text{ kips}}{\frac{\pi (1.6 \text{ in})^2}{4}} = 6.7 \text{ ksi (T)}$$

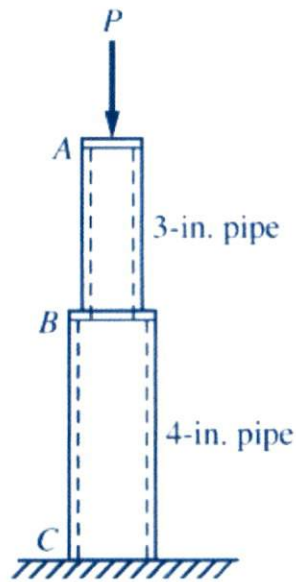
46 MPa

$$\sigma_{CD} = \frac{+4.5 \text{ kips}}{\frac{\pi (0.8 \text{ in})^2}{4}} = 8.9 \text{ ksi (T)}$$

61 MPa

9-6

A short column composed of two standard steel pipes is subjected to a load $P = 20$ kips, as shown in Fig. P9-6. Determine the compressive stress in each pipe. Neglect the weight of the pipes.



From Table A-5(a)

$$3'' \text{ steel Pipe } A = 2.23 \text{ in}^2$$

$$4'' \text{ steel Pipe } A = 3.17 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{-20 \text{ kips}}{2.23 \text{ in}^2} = -8.97 \text{ ksi (c)}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{-20 \text{ kips}}{3.17 \text{ in}^2} = -6.31 \text{ ksi (c)}$$

9-7

Determine the size steel rod, to the nearest sixteenth of an inch, needed to support a tensile load of 40 kips if the allowable tensile stress of steel is 22 ksi.



$$\sigma_{\text{allow}} = 22 \text{ ksi}$$

Normal Stress

$$\sigma_{\text{allow}} = \frac{P}{A}$$

$$A = \frac{P}{\sigma_{\text{allow}}} = \frac{40 \text{ kips}}{22 \text{ kips/in}^2} = 1.8181 \text{ in}^2$$

Circular Rod

$$A = \frac{\pi d^2}{4}$$

$$\frac{\pi d^2}{4} = 1.8181 \text{ in}^2$$

$$d = \sqrt{\frac{4(1.8181 \text{ in}^2)}{\pi}} = 1.521506 \text{ in.}$$

use, $d = \underline{\underline{1 \frac{9}{16} \text{ in.}}}$

$$.521506 \times 16 = 8.34$$

use $\frac{9}{16}$

9-8

Determine the size of steel rod, to the nearest in., needed to support a tensile load of 45 kip if the allowable tensile stress of steel is 22 ksi.

Solution.



$$\sigma_{\text{allow}} = 22 \text{ ksi}$$

Normal Stress

$$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma_{\text{allow}}} = \frac{45 \text{ kip}}{22 \text{ kip/in}^2} = 2.04545 \text{ in}^2$$

Circular Rod,

$$A = \frac{\pi d^2}{4}$$

$$\frac{\pi d^2}{4} = 2.04545 \text{ in}^2$$

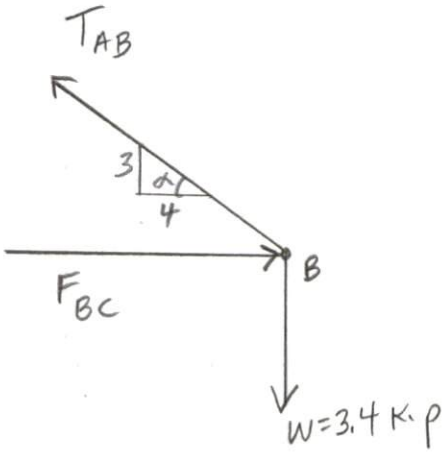
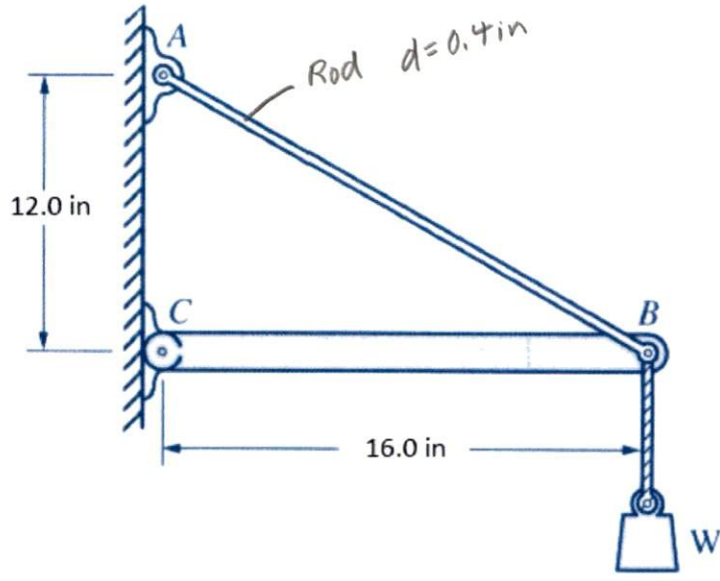
$$d = \sqrt{\frac{4(2.04545 \text{ in}^2)}{\pi}} = 1.6138 \text{ in}$$

use, $d = 2.0 \text{ in}$

9-9

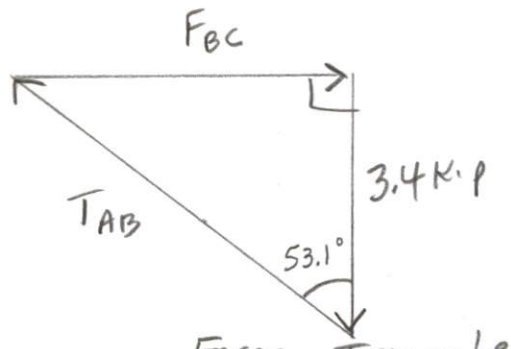
If rod AB in Fig. P9-9 has a diameter of 0.4 in., determine the normal stress in the rod due to a weight $W = 3.4$ kip.

Solution.



FBD

$$\alpha = \tan^{-1} \frac{3}{4} = 36.9^\circ$$



Force-Triangle

$$\cos 53.1^\circ = \frac{3.4 \text{ kip}}{T_{AB}}$$

$$T_{AB} = \frac{3.4 \text{ kip}}{\cos 53.1^\circ} = 5.67 \text{ kips}$$

Normal Stress

$$\begin{aligned} \sigma_{AB} &= \frac{P}{A} = \frac{T_{AB}}{A} = \frac{5.67 \text{ kips}}{\frac{\pi (0.4 \text{ in.})^2}{4}} \\ &= \underline{\underline{45 \text{ ksi (T)}}} \end{aligned}$$

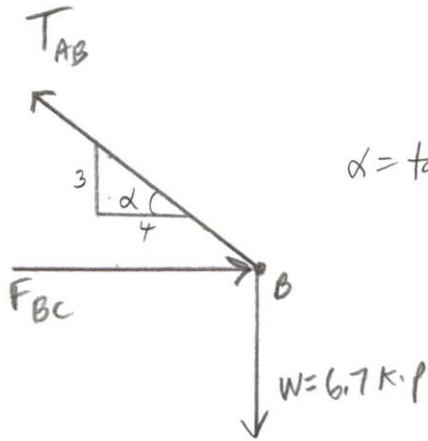
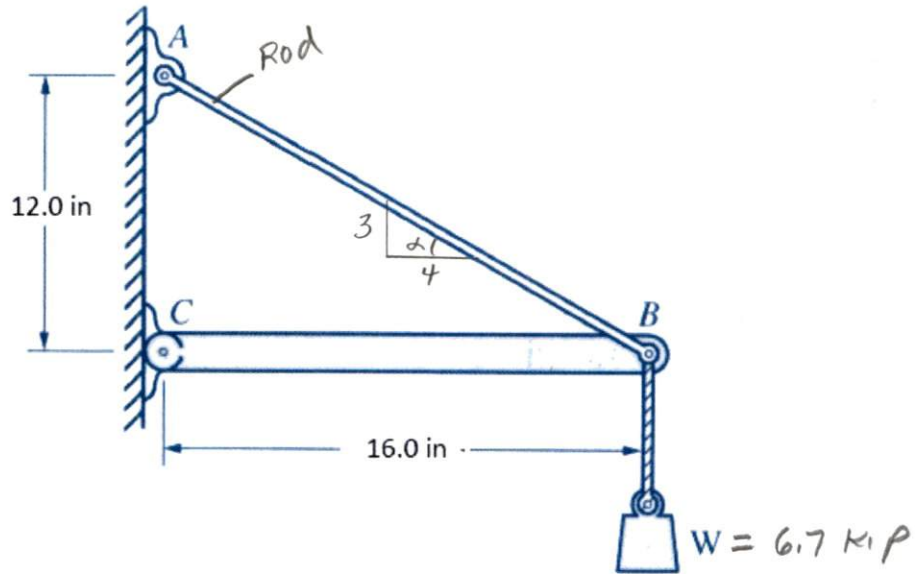
Compare to textbook ans
318 MPa (T)
45 ksi = 310 MPa ✓

9-10

Refer to Fig. P9-9. Determine the diameter of the rod AB, to the nearest in., needed to support a weight $w = 6.7$ kip and the allowable tensile stress is 22 ksi.

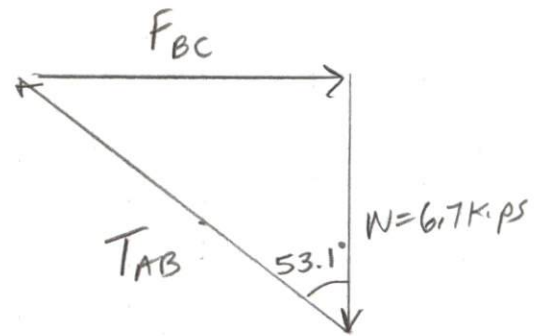
Solution.

Rod
 $\sigma_{allow} = 22 \text{ ksi}$



FBD

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$$



Force-Triangle

$$\cos 53.1^\circ = \frac{6.7 \text{ kips}}{T_{AB}}$$

$$T_{AB} = \frac{6.7 \text{ kips}}{\cos 53.1^\circ} = 11.16 \text{ kips}$$

Circular Rod,

$$A = \frac{\pi d^2}{4}$$

$$\sigma_{allow} = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma_{allow}} = \frac{11.16 \text{ kips}}{22 \text{ kips/in}^2} = 0.5072 \text{ in}^2$$

$$\frac{\pi d^2}{4} = 0.5072 \text{ in}^2$$

$$d = \sqrt{\frac{4(0.5072 \text{ in}^2)}{\pi}} = 0.803625 \text{ in}$$

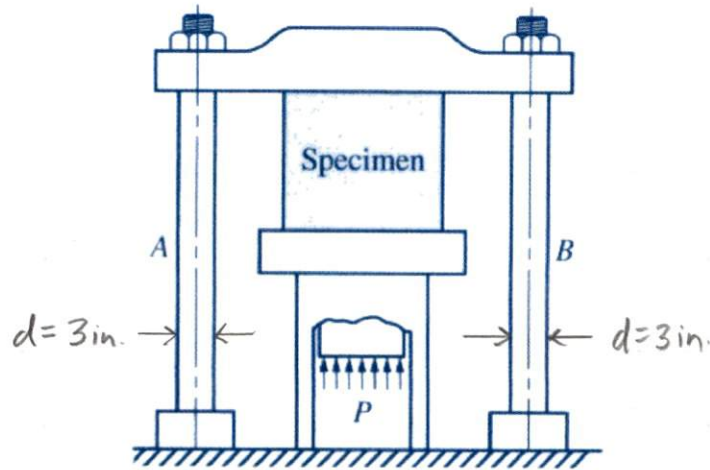
use, d = 1.0 in

9-13

Determine the maximum hydraulic compression, in lb, that can be applied to the testing machine in Fig. P9-13. Each of the two posts, A and B, has a diameter $d = 3$ in. and an allowable tensile stress of 29 ksi.

Solution.

$$\sigma = \frac{P}{A}$$
$$P = A\sigma$$



cross-sectional Area of the post

$$A = \frac{\pi d^2}{4} = \frac{\pi (3 \text{ in})^2}{4} = 7.06858 \text{ in}^2$$

$$\sigma_{\text{allow}} = 29 \text{ ksi}$$

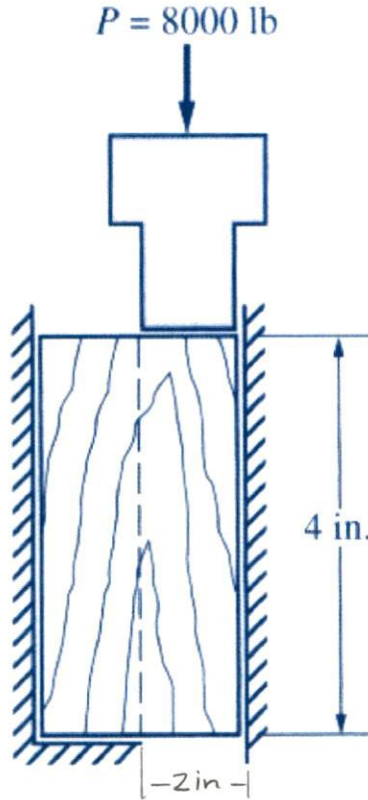
(Two posts)

$$P = 2A \sigma_{\text{allow}} = 2 (7.06858 \text{ in}^2) (29 \text{ Kips/in}^2)$$
$$= 410 \text{ Kips}$$

9-16

A schematic diagram of the apparatus for determining the ultimate shear strength (failure shear stress) of wood is sketched in Fig. P9-16. The test specimen is 4 in. high, 2 in. wide, and 2 in. deep. If the load required to shear the specimen into two pieces is 8000 lb, determine the ultimate shear strength of the specimen.

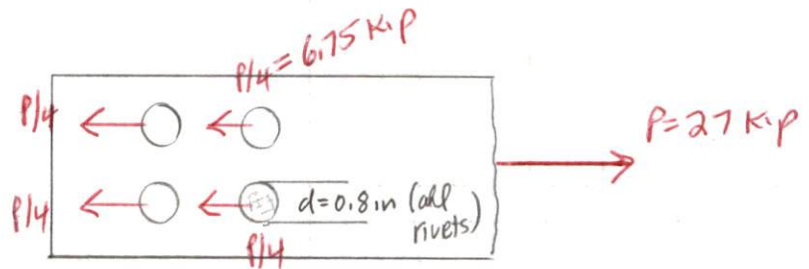
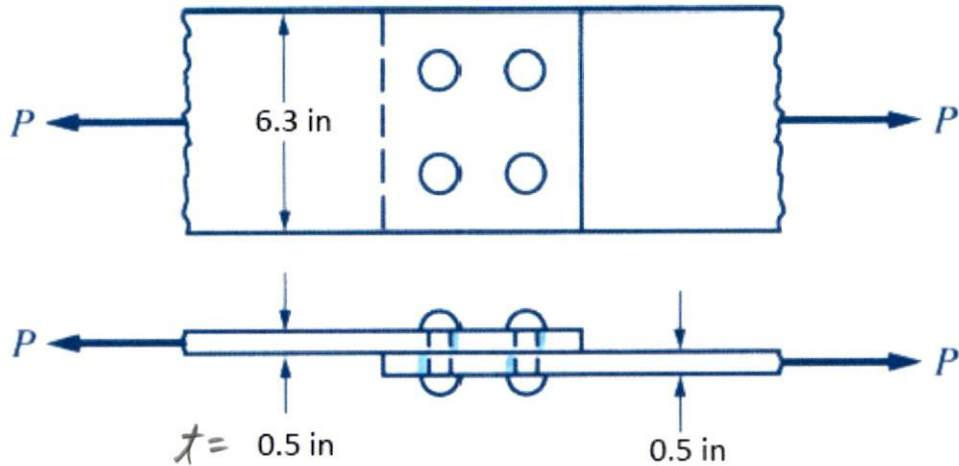
Solution.



$$\tau = \frac{P}{A_s} = \frac{8000 \text{ lb}}{(2 \text{ in})(4 \text{ in})} = \underline{\underline{1000 \text{ psi}}}$$

9-17

The lap joint shown in Fig. P9-17 is connected by four 0.8 in-diameter rivets. Determine (a) the shear stress in the rivets and (b) the bearing stress between the rivets and the plates. Assume that the load $P = 27$ kip carried equally by the four rivets.



a. Shear Stress in the rivets

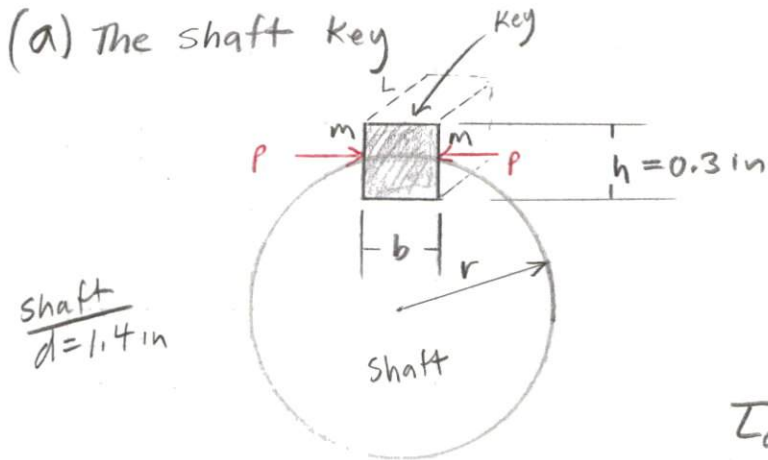
$$\tau = \frac{P}{A_s} = \frac{P/4}{\frac{\pi d^2}{4}} = \frac{27 \text{ kip}}{4} \cdot \frac{4}{\pi (0.8 \text{ in})^2} = 13.4 \text{ ksi}$$

b. Bearing stress between rivets and the plates

The area is approximated by the projection, $t d$

$$\sigma_b = \frac{P}{A_b} = \frac{P}{t d} = \frac{27 \text{ kip}}{4} \cdot \frac{1}{(0.8 \text{ in})(0.5 \text{ in})} = 16.875 \text{ ksi}$$

A 1.4 in-diameter shaft transmits a torque of 700 lb ft by means of a chain drive. The chain sprocket is fastened to the shaft by means of an 0.3 in x 0.3 in square key 2.0 in long. Determine (a) the shear stress in the key and (b) the bearing stress between the key and the shaft.



Moment about the center of the shaft
 $P r = M$ (transmitted moment)

$$P = \frac{M}{r}$$

$$\tau_{avg} = \frac{P}{A_s} = \frac{M}{r} = \frac{M}{r b L}$$

$$b = 0.3 \text{ in} \quad (\text{width of the key})$$

$$L = 2.0 \text{ in} \quad (\text{length of the key})$$

$$r = \frac{1.4}{2} \text{ in} \quad (\text{radius of the shaft})$$

$$M = 700 \text{ lb}\cdot\text{ft} \quad (\text{torque})$$

$$\tau_{avg} = \frac{M}{r b L} = \frac{700 \text{ lb}\cdot\text{ft} \left(\frac{12 \text{ in}}{\text{ft}} \right)}{0.7 \text{ in} (0.3 \text{ in}) (2.0 \text{ in})} = \frac{8400 \text{ lb}\cdot\text{in}}{0.42 \text{ in}^3}$$

$$= 20,000 \text{ lb/in}^2$$

$$= \underline{\underline{20 \text{ Ksi}}} \quad (138 \text{ MPa})$$

(b) Bearing stress between the key and the shaft

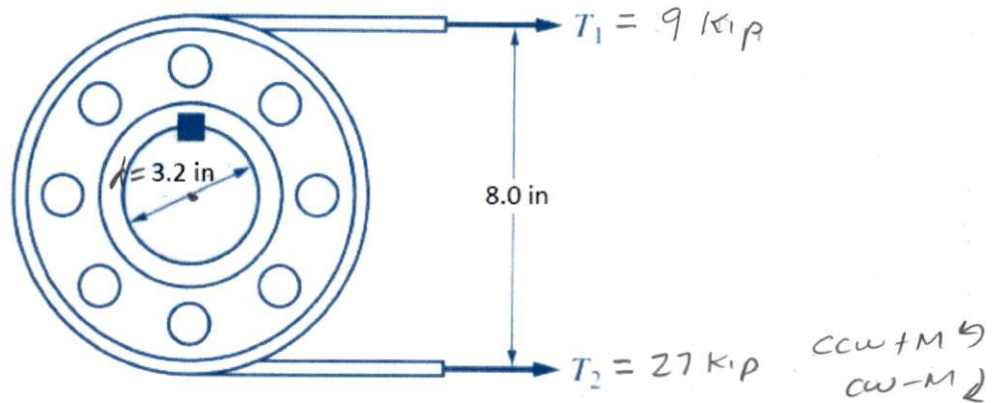
$$A_b = \frac{h}{2} (L) = \frac{0.3 \text{ in}}{2} (2.0 \text{ in}) = 0.3 \text{ in}^2$$

Bearing stress,

$$\sigma_b = \frac{P}{A_b} = \frac{M}{r} = \frac{700 \text{ lb}\cdot\text{ft} \left(\frac{12 \text{ in}}{\text{ft}} \right)}{0.7 \text{ in}} = \frac{8400 \text{ lb}}{0.3 \text{ in}^2} = \underline{\underline{40 \text{ Ksi}}} \quad (276 \text{ MPa})$$

9-20

The pulley shown in Fig. P9-20 is connected to a 3.2 in-diameter shaft by a 0.8 in square key that is 4.0 in long. If the belt tensions are $T_1 = 9$ kip and $T_2 = 27$ kip, determine (a) the shear stress in the key and (b) the bearing stress between the key and the shaft.

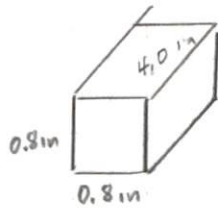


(a) Key (square)

$$b = 0.8 \text{ in.}$$

$$h = 0.8 \text{ in.}$$

$$L = 4.0 \text{ in.}$$



Shaft

$$d = 3.2 \text{ in}$$

$$r = \frac{d}{2} = 1.6 \text{ in.}$$

Transmitted Moment

$$M = T_2 (r_{\text{pulley}}) - T_1 (r_{\text{pulley}})$$

$$= (T_2 - T_1) r_{\text{pulley}}$$

$$= (27 \text{ kip} - 9 \text{ kip}) (4.0 \text{ in})$$

$$= 72 \text{ kip}\cdot\text{in}$$

$$P = \frac{M}{r} = \frac{72 \text{ kip}\cdot\text{in}}{1.6 \text{ in.}}$$

$$= 45 \text{ kip}$$

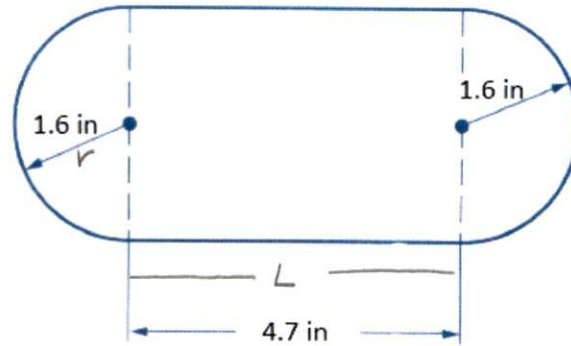
Shear Stress (Key)

$$\tau = \frac{P}{A_s} = \frac{45 \text{ kip}}{0.8 \text{ in} (4.0 \text{ in})} = 14.0625 \text{ ksi}$$

$$(b) \quad \sigma_b = \frac{P}{A_b} = \frac{45 \text{ kip}}{\left(\frac{0.8 \text{ in}}{2}\right) (4.0 \text{ in})} = 28.125 \text{ ksi} \quad (\text{Bearing Stress Key and shaft})$$

9-22

Determine the minimum force that must be exerted on a punch to shear a hole, having the shape shown in Fig. P9-22, through a steel plate 0.16 in thick. The plate has an ultimate shear strength (failure shear stress) of 44 ksi.



$$r = 1.6 \text{ in}$$

$$L = 4.7 \text{ in}$$

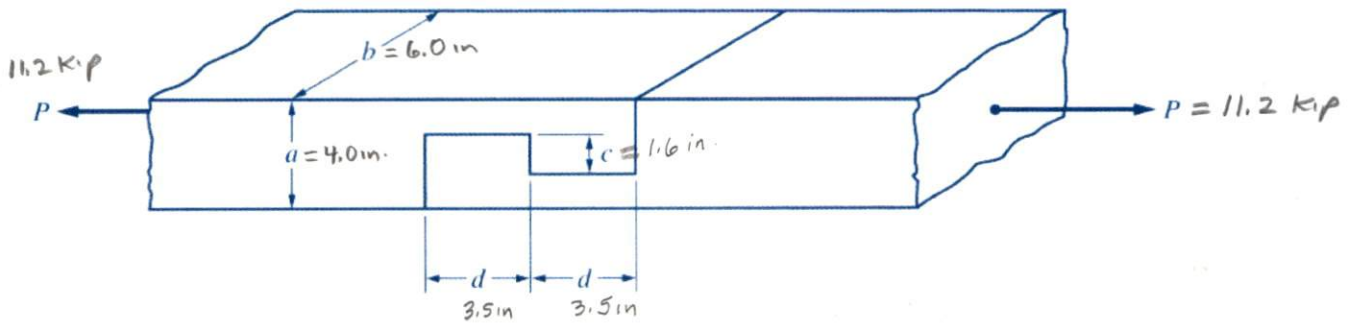
$$t = 0.16 \text{ in}$$

$$\begin{aligned} A_s &= (2\pi r + 2L)t \\ &= [2\pi(1.6 \text{ in}) + 2(4.7 \text{ in.})](0.16 \text{ in.}) \\ &= 3.112495 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} P_{\min} &= A_s \tau_u = 3.112495 \text{ in.}^2 (44 \text{ kip/in.}^2) \\ &= 137 \text{ kip} \end{aligned}$$

9-24

The dimensions in the wood joint shown in Fig. P9-24 are $a = 4.0$ in, $b = 6.0$ in, $c = 1.6$ in, and $d = 3.5$ in. Determine the shear stress and the bearing stress in the joint if $P = 11.2$ kip.



Shear Stress

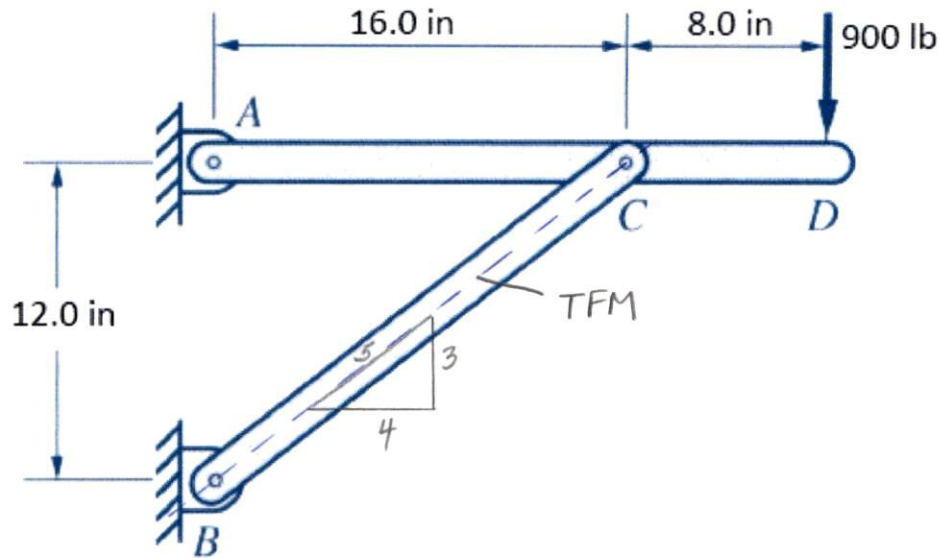
$$\tau = \frac{P}{A_s} = \frac{11.2 \text{ kip}}{3.5 \text{ in} (6.0 \text{ in})} = 0.533 \text{ ksi}$$

Bearing Stress

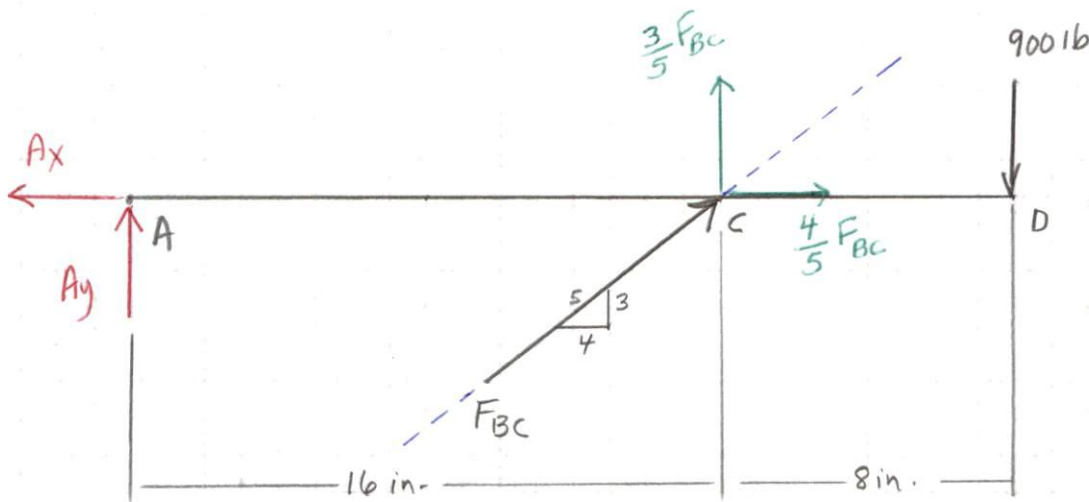
$$\sigma_b = \frac{P}{A_b} = \frac{11.2 \text{ kip}}{(1.6 \text{ in}) (6.0 \text{ in})} = 1.167 \text{ ksi}$$

9-26

The structure shown in Fig. P9-26 is fastened to the support by bolts at A and B. The bolts are in double shear. If the allowable shear stress in the bolts is 17.4 ksi select the sizes of the bolts at A and B.



Member ACD



FBD - member ACD

ccw + M ↺
cw - M ↻

Equilibrium Equations

$$[\sum M_A = 0] \quad \frac{3}{5} F_{BC} (16 \text{ in.}) - 900 \text{ lb} (24 \text{ in.}) = 0$$

$$F_{BC} = \frac{\frac{5}{3} (21,600 \text{ lb} \cdot \text{in.})}{16 \text{ in.}} = 2250 \text{ lb (c)}$$

$$[\sum F_x = 0] \quad -A_x + \frac{4}{5} F_{BC} = 0$$

$$A_x = \frac{4}{5} (2250 \text{ lb}) = \underline{\underline{1800 \text{ lb} \leftarrow}}$$

$$[\sum F_y = 0] \quad A_y + \frac{3}{5} F_{BC} - 900 \text{ lb} = 0$$

$$A_y = 900 \text{ lb} - \frac{3}{5} (2250 \text{ lb}) = -450 \text{ lb} \uparrow$$

$$\text{and } \boxed{A_y = 450 \text{ lb} \downarrow}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = 1855 \text{ lb}$$

Bolt at A is in double shear. Each area carries one-half the load.

$$A_s = \frac{\frac{R_A}{2}}{\tau_{\text{allow}}} = \frac{\frac{1855 \text{ lb}}{2} \left(\frac{\text{kip}}{1000 \text{ lb}} \right)}{17.4 \frac{\text{kip}}{\text{in}^2}} = 0.0533 \text{ in.}^2$$

$$\frac{\pi d^2}{4} = 0.0533 \text{ in.}^2$$

$$d = \sqrt{\frac{4(0.0533 \text{ in.}^2)}{\pi}} = 0.26052 \text{ in.}$$

use, $d = 5/16 \text{ in}$ (pin @ A)

Bolt at B

$$R_B = F_{BC} = 2250 \text{ lb}$$

$$A_s = \frac{\frac{R_B}{2}}{\tau_{\text{allow}}} = \frac{\frac{2250 \text{ lb}}{2} \left(\frac{\text{kip}}{1000 \text{ lb}} \right)}{17.4 \frac{\text{kip}}{\text{in}^2}} = 0.06465 \text{ in.}^2$$

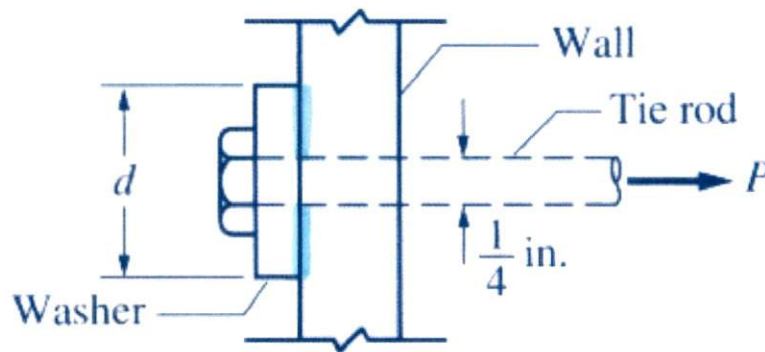
$$\frac{\pi d^2}{4} = 0.064565 \text{ in.}^2$$

$$d = \sqrt{\frac{4(0.064565 \text{ in.}^2)}{\pi}} = 0.287 \text{ in.}$$

use, $d = 6/16 \text{ in}$ (pin @ B)

9-27

See Fig. P9-27. A tie rod of $\frac{1}{4}$ -in. diameter is used to hold a plaster wall in place. The tensile stress in the rod caused by P is 20 ksi. Find the diameter d of the washer that keeps the bearing stress between the plaster and the washer from exceeding 300 psi.



Tie Rod

$$\sigma = \frac{P}{A} \Rightarrow P = \sigma A = 20 \frac{\text{Kip}}{\text{in}^2} \left(\frac{\pi (0.25 \text{ in.})^2}{4} \right) = 0.9817 \text{ Kip}$$

$$= 982 \text{ lb}$$

Bearing Area

$$A_b = \text{Area Washer} - \text{Area Tie rod} = \frac{\pi d^2}{4} - \frac{\pi (0.25 \text{ in.})^2}{4}$$

$$A_b = \frac{P}{\sigma_{b \text{ allow}}} = \frac{982 \text{ lb}}{300 \text{ psi}} = 3.27 \text{ in.}^2$$

$$\frac{\pi d^2}{4} - \frac{\pi (0.25 \text{ in.})^2}{4} = 3.27 \text{ in.}^2$$

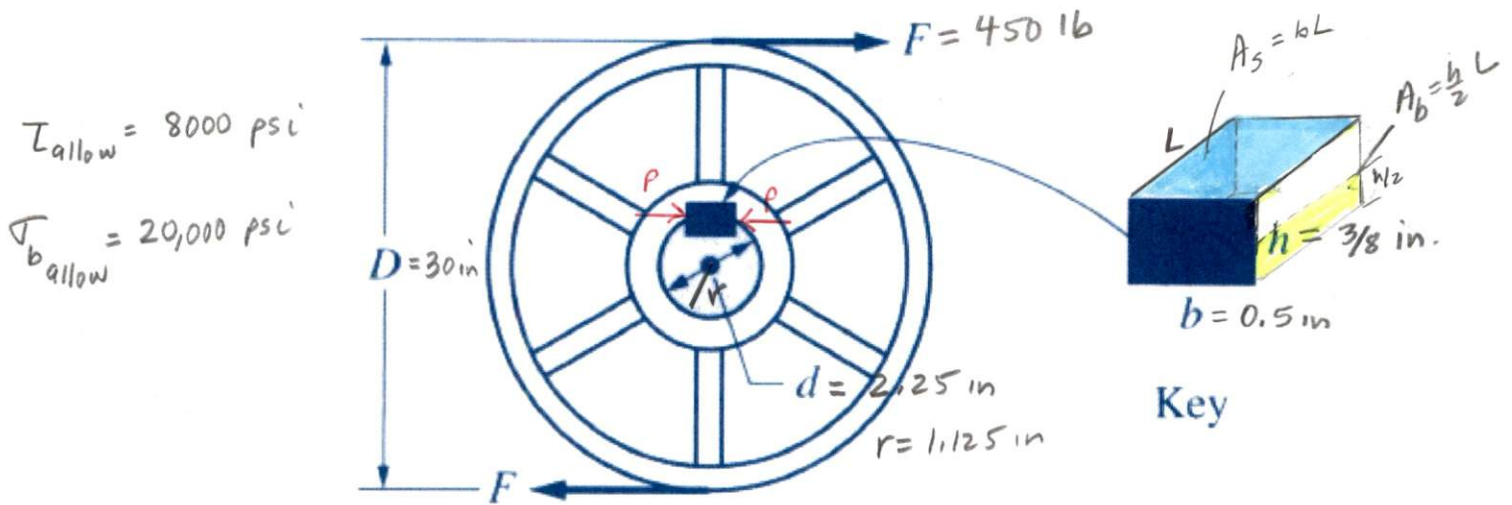
$$\frac{\pi}{4} (d^2 - (0.25 \text{ in.})^2) = 3.27 \text{ in.}^2$$

$$d = \sqrt{\frac{4}{\pi} (3.27 \text{ in.}^2) + (0.25 \text{ in.})^2} = 2.056 \text{ in.}$$

$$\text{use, } d = \underline{\underline{2 \frac{1}{16} \text{ in.}}} \quad (2.0625 \text{ in.})$$

9-28

The control gate in Fig. P9-28 is operated by a wheel and shaft connected by a flat key, as shown. The allowable stresses in the key are 8000 psi in shear and 20 000 psi in bearing. If $d = 2\frac{1}{4}$ in., $D = 30$ in., $b = \frac{1}{2}$ in., $h = \frac{3}{8}$ in., and $F = 450$ lb, determine the length of the key.



Moment Transmitted Due to the Couple

$$M = 450 \text{ lb} (30 \text{ in}) = 13,500 \text{ lb} \cdot \text{in.}$$

$$P = \frac{M}{r} = \frac{13,500 \text{ lb} \cdot \text{in.}}{1.125 \text{ in.}} = 12000 \text{ lb}$$

Must check Both shear stress and Bearing stress

Shear stress

$$A_s = \frac{P}{\tau_{allow}} = \frac{12000 \text{ lb}}{8000 \text{ lb/in}^2} = 1.5 \text{ in}^2$$

$$A_s = bL = (0.5 \text{ in}) L = 1.5 \text{ in}^2$$

$$L = 3.0 \text{ in}$$

Bearing stress

$$A_b = \frac{P}{\sigma_{b,allow}} = \frac{12000 \text{ lb}}{20000 \text{ lb/in}^2} = 0.6 \text{ in}^2$$

$$A_b = \frac{h}{2} L = \frac{0.375 \text{ in}}{2} L = 0.6 \text{ in}^2$$

$$L = \underline{\underline{3.2 \text{ in}}}$$

Bearing stress governs design. Use key length, $L = 3\frac{1}{4} \text{ in}$
(3.25 in)